Technical Notes

Remarks and an Experiment on Magnetogasdynamic Flow about a Sphere

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Correlation Parameter for MHD Flows with Transverse Fields

THE macroscopic magnetohydrodynamic equation of motion for an electrically conducting incompressible medium in which the Hall-current effect and the ion-slip effect are neglected and in which the current loops close on themselves is nondimensionalized. Thus

$$D\mathbf{V}/dt + \text{grad } p = (1/R)\cdot\nabla^2\mathbf{V} + S\cdot\mathbf{j}\times\mathbf{B} \tag{1}$$

in the usual notation. For scalar conductivity, the $\mathbf{j}\times\mathbf{B}$ force has a component directed oppositely to $\mathbf{V}$, of order σVB^2, for the low magnetic Reynolds number case

$$R = \frac{V_0 l_0}{\nu_0} \quad \text{represents the ratio of forces per unit volume} \left\{\frac{\text{inertia}}{\text{viscous}}\right\} \tag{2}$$

$$S = \frac{\sigma_0 B_0{}^2 l_0}{\rho_0 V_0} \quad \text{represents the ratio of forces per unit volume} \left\{\frac{\mathbf{j}\times\mathbf{B}}{\text{inertia}}\right\} \tag{3}$$

If $1/R$ and S are large, then the inertia terms on the left-hand side of Eq. (1) can be neglected, and another nondimensional parameter $(R\cdot S)$ appears; the $[R\cdot S]^{1/2}$ usually is called the Hartmann number H_M.

$$H_M{}^2 = \frac{\sigma_0 B_0{}^2 l_0}{\rho_0 \nu_0} \quad \text{represents the ratio of forces per unit volume} \left\{\frac{\mathbf{j}\times\mathbf{B}}{\text{viscous}}\right\} \tag{4}$$

The work of Lundquist[1] and Lehnert[2] for uniform transverse field configurations indicates that the variation of the velocity is confined to the boundary region $L_M = [V_0\rho_0/\sigma_0 B_0{}^2]^{1/2}$ and suggests that the length dimension entering the ordinary fluid mechanical Reynolds number be replaced by L_M:

$$R_H = \frac{V_0}{\nu_0}\cdot L_M = \left[\frac{V_0 l_0}{\nu_0}\cdot\frac{\rho_0 V_0}{\sigma_0 B_0{}^2 l_0}\right]^{1/2} = \left[\frac{R}{S}\right]^{1/2} \tag{5}$$

The implication of Lundquist's suggestion is that an indication of how the magnetohydrodynamics (MHD) vorticity interaction problem scales with an $S \approx 0(1)$ or $RS \gg 0(1)$ may be obtained by replacing the usual fluid mechanical Reynolds number of the zero-field case with the Hartmann-layer Reynolds number $R_H = [R/S]^{1/2}$. This broad role for the number R_H prompts one to search for a general, though perhaps imprecise, deduction of R_H from the equations of motion. Equation (1) might be characterized as follows:

$$\{\text{inertia terms}\} = \left[\frac{1}{R}\left(\frac{\text{viscous forces}}{\text{inertia forces}}\right)\right]\cdot\{\text{viscous terms}\} + \left[S\left(\frac{\mathbf{j}\times\mathbf{B}\text{ forces}}{\text{inertia forces}}\right)\right]\cdot\{\mathbf{j}\times\mathbf{B}\text{ terms}\} \tag{6}$$

In ordinary incompressible viscous fluid mechanics, most of the effects scale with the ratio of the first power of driving force to retarding force, i.e., R. In the combined MHD vorticity interaction case, one can make a similar number in two ways: 1) an arithmetic combination, or 2) a geometric combination R^q/S^{1-q}, where q is an integer between 0 and 1. Table 1 lists three geometric combinations.

It is curious that, although the correlating parameter R_H has appeared in an important way in both theoretical and experimental investigations, it has not been singled out as such. The $q = \frac{1}{2}$ case has usually been written in terms of the Hartmann number $R_H = H_M/S$. This seems to obscure its fundamental nature as the ratio

$$R_H = \left[\frac{R}{S}\right]^{1/2} = \left[\frac{\text{inertia}}{\text{viscous}}\cdot\frac{\text{inertia}}{\mathbf{j}\times\mathbf{B}}\right]^{1/2} = \left[\frac{\text{driving force}}{\text{retarding force}}\right]^{1} \tag{7}$$

Table 1 Three geometric combinations

Type of fluid dynamics	$RS = \left\{\frac{\mathbf{j}\times\mathbf{B}\text{ forces}}{\text{viscous forces}}\right\}$	q	$(R^q/S^{1-q}) = \left\{\frac{\text{driving force}}{\text{retarding force}}\right\}$
Ordinary viscous	0	1	R
Combined viscous magneto	0(1)	$\frac{1}{2}$	R_H
Ordinary inviscid magneto	∞	0	$1/S$

Incompressible Experiment

In this discussion, S is based on the assumption that the velocity perpendicular to B is of the order of the freestream velocity. In the sphere base flow with field aligned to the flight direction, the base vortex ring has velocities transverse to the field, which are of the order of the freestream velocities.

Maxworthy[3] performed a series of experiments in which metal spheres of several different diameters fell vertically through an electrically conducting fluid (liquid sodium) and a solenoid magnetic field. Now, according to Lundquist's suggestion, one expects the onset of wake unsteadiness to occur in the MHD sphere wake at the value of $R_H[= (R/$

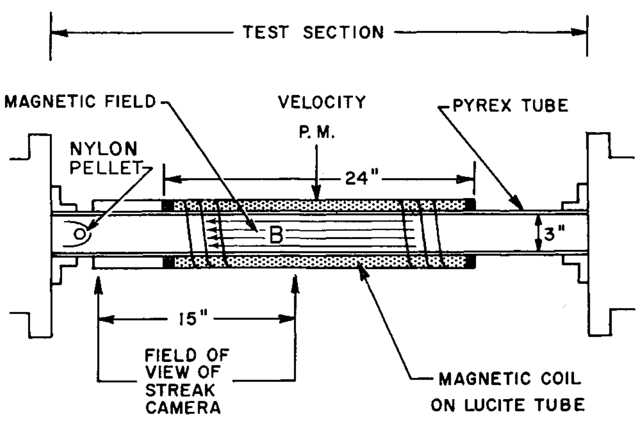

Fig. 1 Schematic of hypersonic MHD base experiment.

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$S)^{1/2}]$ equal to that value of R at which it occurs in the zero-field case. The incompressible wake experiments of Margarvey and Bishop, as analyzed by Fay and Goldburg,[4] yield for onset of wake unsteadiness (R based on diameter)

$$R_{\text{crit}}\big|_{B=0} \approx 200 \tag{8}$$

Thus, the prediction in the MHD case for $S = 0(1)$ or $RS \gg 0(1)$ is

$$R_H\big|_{\text{crit}} \approx 200 \tag{9}$$

Maxworthy's experiment,[3] as analyzed by Goldburg,[5] tends to confirm this. For the test conditions for 11 flights, the $R_H|_{\text{crit}}$ for onset of wake unsteadiness appears to be between 111 and 143.

Hypersonic Magnetogasdynamic Wake Experiments

For compressible flow, it is usual to extend the incompressible result to the compressible case with the qualification that the representative local values, rather than freestream values, are used. Values characteristic of the base-region conditions are taken as representative for the present purposes.

A hypersonic counterpart of Maxworthy's incompressible experiment is a ballistic range experiment in which pellets fly through an aligned solenoid magnetic field. Evaluation of the relevant MHD parameters in the base region of hypersonic pellets was made. Argon gas was taken to be the working fluid to maximize gas luminosity and electrical conductivity. An assumption of presence of easily ionizable ablation products was made to give Spitzer conductivity. Calculations for the base-region values for the parameters S/B^2l, H_M/Bl, $R_H \cdot B$, $\omega\tau/B$, and R_M/l are presented in Ref. 5. The Hall coefficient $\omega\tau$ is a measure of the angle between the $\mathbf{V} \times \mathbf{B}$ vector and the current vector due to tensor conductivity.

Fay and Goldburg[4] indicate that transition from laminar to turbulent wakes behind 0.22-in.-diam spheres in argon at velocities of 14,000 fps, e.g., occurs at a Reynolds number based on shoulder conditions of 2000. The corresponding base-region value is about 600. The corresponding ambient pressure is about 1-cm Hg.

The suggested criterion for suppression of wake transition is that the fluid mechanical Reynolds number based on Hartmann-layer thickness R_H be less than the ordinary transition fluid mechanical Reynolds number $R_{X,0}$. For a range pressure of 5-cm Hg and velocities of about 14,000 fps, this condition is met when the magnetic field strength B is chosen to make $\omega\tau = 1.0{:}R_H \approx 75 < R_{X,0} \approx 600$; $[S \approx 0(1),\ RS \approx 0(10^3)]$, fulfilling the requirements for an MHD vorticity interaction. A set of the calculations of the MHD parameters, assuming that the gas in the base region is ionized argon alone, is contained in Ref. 5. These calculations indicate that an MHD vorticity interaction is unlikely in the ballistic range experiment if the gas properties are assumed to be caused solely by ionized argon; for pure argon with the same ballistic range conditions, $R_H|_{\omega\tau=1.0} > 1000$.

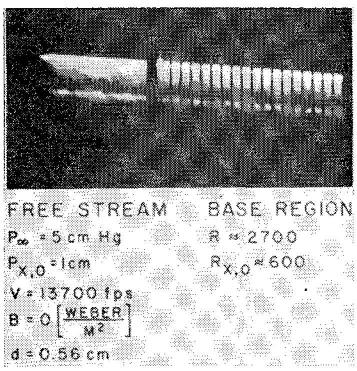

Fig. 2 Turbulent drum camera photograph, argon gas, without MHD, First Set (the initial vertical break in the streak photograph is the downstream coil support face plate; the subsequent vertical breaks are the wire turns of the solenoid).

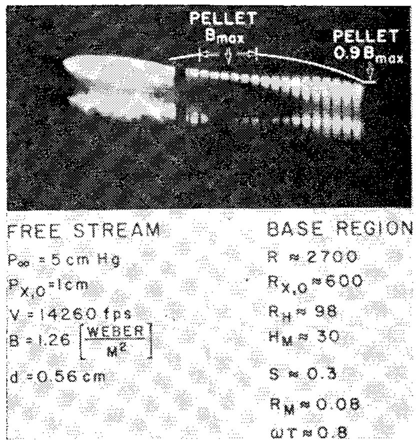

Fig. 3 Turbulent drum camera photograph, argon gas, with MHD, First Set (the pellet position at which the maximum magnetic field strength is reached is shown with the band of uncertainty; the lower image, which runs parallel to the pellet in the figure, is a refocusing of the shock off the test section walls).

The experiment to explore the possibility of suppressing hypersonic wake transition is shown in Fig. 1. Nylon pellets 0.22 in. in diameter are fired at velocities near 14,000 fps through the magnetic field of a solenoid placed about a glass test section in the Avco-Everett Research Laboratory (AERL) ballistic range. The magnetic field of the solenoid is produced by passing a current through the wire by the discharge of a capacitor bank. The use of streak-drum camera photographs to differentiate between laminar and turbulent wakes[4] has been reported previously. The streak camera is oriented so that its field of view encompasses a region before and including part of the coil. The appearance of luminous structure in drum camera photographs is interpreted as turbulent wake flow, whereas smooth and diffuse luminosity is interpreted as laminar flow. The streak photographs of the pellet within the coil provide sufficient information to reveal the absence or presence of luminous turbulent structure in the wake (Figs. 2 and 3).

A streak photograph with the coil energized is shown in Fig. 3. Such a photograph was obtained in each of the eight First Set runs. Turbulent luminous streaks are evident in the region before the solenoid where the magnetic field strength is vanishingly small. As the pellet enters the magnetic field of the coil, a definite change in the structure of the luminous flow field occurs. The luminous streaks, which characterize the turbulent flow[4] diminish and disappear, and a region, which is smooth and devoid of streaks, appears. From the information obtained in the First Set of runs, it appears that the originally turbulent wake flow is substantially affected by the presence of the magnetic field, perhaps suggestive of a return to a laminar wake flow.

The Second Set of runs took place in a clinically clean range in contradistinction to the First Set, which took place in a range, a part of which may have been coated with solid sodium because runs for a sodium schlieren experiment[6] were being shot alternately with the First Set wake-MHD runs. In the Second Set of runs, the effect on wake luminosity did not reappear. As pointed out previously, the assumption of easily ionizable particles leading to a Spitzer conductivity (the largest possible) provides $R_H < R_{X,0}$, whereas the lack of such particles leads to conductivities that provide $R_H >$

$R_{X,0}$. Calculations and available data show that the number of sodium dust particles required to produce the Spitzer concentration of electrons in the base region could have been present during the First Set of runs.[5]

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Strouhal Numbers for the Hypersonic Wakes of Spheres and Cones

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Introduction

Fay and Goldburg have presented evidence of the fluid mechanical similarity of the unsteady hypersonic wake and the unsteady incompressible wake.[1] The Strouhal number ($S = fL/V$ where V is the freestream velocity, f is the wake periodic frequency, and L is a characteristic length) is a characterization of the prominent large scale (wavelength) frequency in the unsteady wake. Recently it was found that the Strouhal number variation for the incompressible wake for a range of cones and spheres could be correlated using a Reynolds number based on total wake momentum thickness as the characteristic length.[2] This paper presents a similar finding for the unsteady hypersonic wake.

Goldburg and Florsheim[2] investigated the unsteady incompressible wake behind various three-dimensional shapes by dropping small bodies into a tank full of an aqueous glycerin solution and observing their wakes. The wakes were rendered visible by a dye from the body as it passed through the fluid. Equation (1) defines the total wake momentum thickness θ:

$$\rho V^2 \pi^j \theta^{j+1} = \text{drag} = \tfrac{1}{2}\rho V^2 C_D A \qquad \begin{cases} j = 0 \text{ planar} \\ j = 1 \text{ axisymmetric} \end{cases} \qquad (1)$$

If one views the body as a black box of arbitrary shape leaving a given wake, and one asks what length parameter best characterizes the shear flow wake phenomenon produced by the dissipation processes connected with the body, then the answer appears to be the total wake momentum thickness θ or equivalently from Eq. (1), $(C_DA/2\pi)^{1/2}$. The incompressible results of Goldburg and Florsheim tend to support the suggestion that total wake momentum thickness

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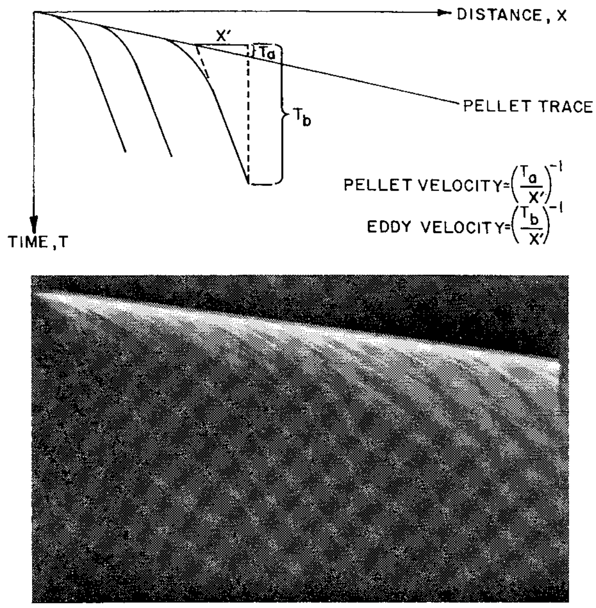

Fig. 1 Typical drum camera streak photograph with the corresponding space-time plot. Run: 10-4-61/2; 6 cm. air; 14,700 fps; 0.22-in. diam., and 15-in. field.

is the appropriate length for the variation of Strouhal number with Reynolds number. It was found that for regular vortex shedding, the data for spheres and a range of cones could be correlated with the Rayleigh Strouhal formula based on θ for the incompressible wake:

$$S_\theta = 0.305[1 - (95/Re_\theta)] \qquad (2)$$

The disk and the needle were deviate cases.[2]

Investigation

Measurements of the frequency of vortex generation behind hypervelocity spheres in air ($M_\infty \simeq 14$) were made from self-luminous drum camera photographs taken in the Avco-Everett Research Laboratory (AERL) ballistic range and in the ballistic range of the Canadian Armament Research and Development Establishment (CARDE) at Valcartier, Quebec.[1] Measurements behind hypervelocity cones at approximately the same Mach number were also made at the CARDE range. In the photographic techniques used, a streak drum camera records the flow field made

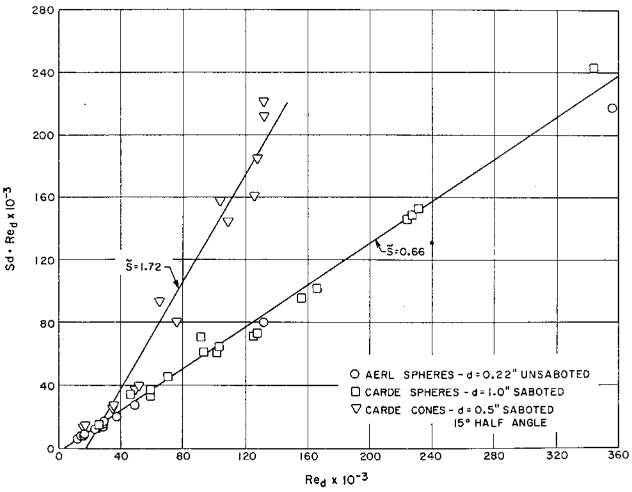

Fig. 2 Strouhal numbers for the hypersonic wake ($M_\infty \approx 14$), spheres and cones; based on body diameter as characteristic length.